

VARIANCE AND COVARIANCE RULES

Let x and y be random variables.

Let a and b be constants.

Covariance Rules

Assume that x and y are not independent. (i.e. there is some dependent relationship between them)

$$\text{cov}(a, b) = 0$$

$$\text{cov}(a, y) = 0$$

$$\text{cov}(x, b) = 0$$

$$\text{cov}(ax, y) = a\text{cov}(x, y)$$

$$\text{cov}(x, by) = b\text{cov}(x, y)$$

$$\text{cov}(ax, by) = abcov(x, y)$$

$$\text{cov}(x+a, y) = \text{cov}(x, y) + \text{cov}(a, y) = \text{cov}(x, y) \text{ because } \text{cov}(a, y) = 0$$

$$\text{cov}(x+a, y+b) = \text{cov}(x, y) + \text{cov}(a, y) + \text{cov}(x, b) + \text{cov}(a, b) = \text{cov}(x, y) \text{ because } \text{cov}(a, y) = \text{cov}(x, b) = \text{cov}(a, b) = 0$$

$$\text{cov}(x, x) = \text{var}(x)$$

$$\text{cov}(ax, ax) = a^2\text{var}(x)$$

[Note: if x and y are independent, then $\text{cov}(x, y) = 0$ since there is no dependent relationship between the two variables]

Variance Rules

$$\text{var}(ax) = a^2\text{var}(x)$$

$$\text{var}(a) = 0$$

$$\text{var}(x + y) = \text{var}(x) + \text{var}(y) + 2\text{cov}(x, y) \text{ [Note: if } x \text{ and } y \text{ are independent, then } \text{var}(x + y) = \text{var}(x) + \text{var}(y)\text{]}$$

$$\text{var}(ax + by) = a^2\text{var}(x) + b^2\text{var}(y) + 2abcov(x, y)$$

NOTE: The sample standard deviation/sample variance can be treated as a constant once it is calculated since it does not change.