ARE 150
Spring 2019 - Week 4
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Labor Supply

People like to consume goods, but they also like to spend time on leisure activities (such as relaxing or communicating with friends on Facebook). However, each individual only has a certain amount of time to allot to their activities in a given day. Typically, people spend some of their time working (supplying labor to the market) and some of their time on leisure. Because there is a tradeoff between spending time at work to earn money so that we can consume goods and spending time on leisure, everyone has to figure out how to split their time between work and leisure. In order to solve for the optimal amount of labor to supply to the farm, we model this problem by using a constrained optimization problem where each individual maximizes their utility subject to an income constraint.

$$\max_{X_l, X_0} \boldsymbol{U} = \boldsymbol{U}(\boldsymbol{X_l}, \boldsymbol{X_0})$$

$$\textbf{\textit{subject to}}:$$

$$\boldsymbol{wX_l} + \boldsymbol{p_0X_0} = \boldsymbol{wT}$$

Where **w** is the wage, X_l is the amount of time spent on leisure, p_0 is the price of other stuff, X_0 is the amount of other stuff, and T is the total amount of time available. The Lagrangean Function that characterizes this problem be expressed as follows:

$$\max_{X_l, X_0} \boldsymbol{L} = \boldsymbol{U}(\boldsymbol{X_l}, \boldsymbol{X_0}) - \boldsymbol{\lambda}(\boldsymbol{wX_l} + \boldsymbol{p_0X_0} - \boldsymbol{wT})$$

Taking the first order conditions (FOCs) of the Lagrangean function gives us the following FOCs.

$$(1) \quad \frac{\partial L}{\partial X_l} = 0 \Leftrightarrow \frac{\partial U}{\partial X_l} - \lambda w = 0$$

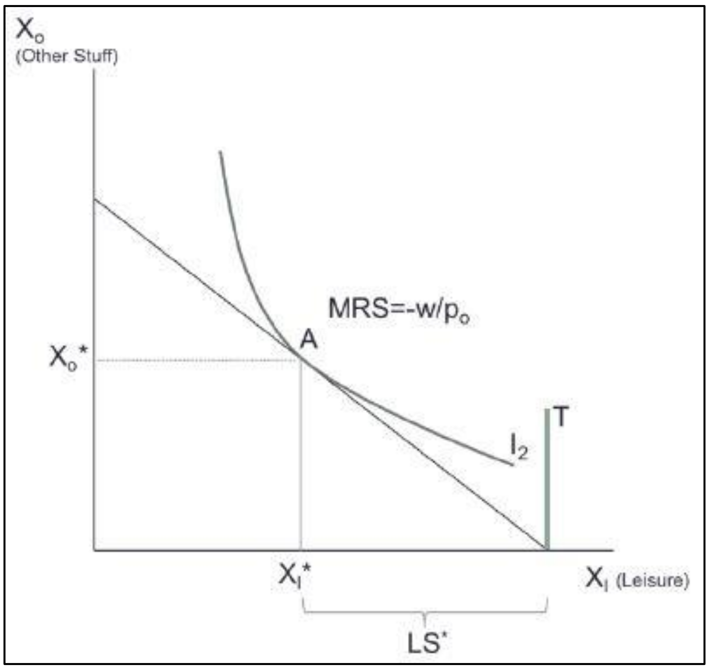
$$(2) \quad \frac{\partial L}{\partial X_0} = 0 \Leftrightarrow \frac{\partial U}{\partial X_o} - \lambda p_0 = 0$$

Taking the ratio of (1)/(2) gives us the following equation:

$$\frac{\frac{\partial U}{\partial X_l}}{\frac{\partial U}{\partial X_0}} = \frac{w}{p_0} \Rightarrow$$

$$(3) \qquad -\frac{\frac{\partial U}{\partial X_l}}{\frac{\partial U}{\partial X_0}} = MRS_{l,0} = -\frac{w}{p_0}$$

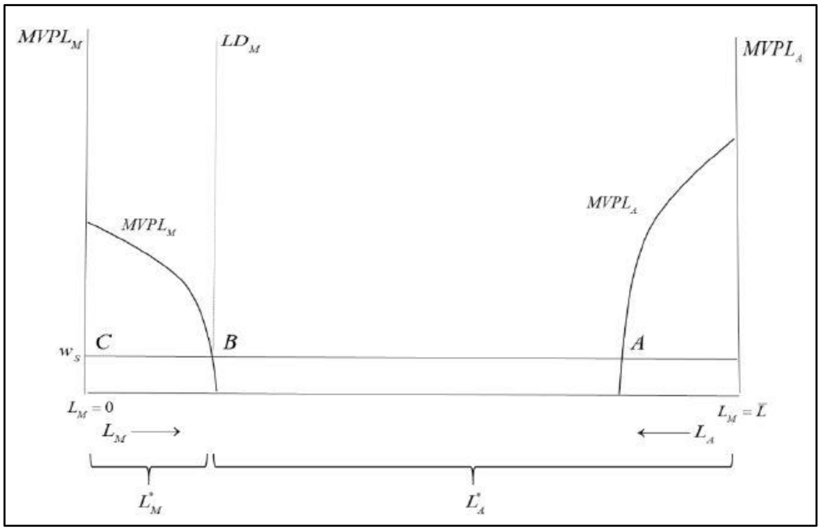
Equation (3) characterizes the condition that must be met in order for the individual to optimize her utility subject to the budget constraint. It says that if the individual is optimizing her welfare, then she will consume at a point where her marginal rate of substitution is just equal to the negative of the ratio of the prices of the goods that she is consuming (recall she is consuming both leisure and "other" stuff). Put differently, in order to maximize her welfare, she will consume at a point where her indifference curve is just tangent to the budget constraint as depicted in the picture below.



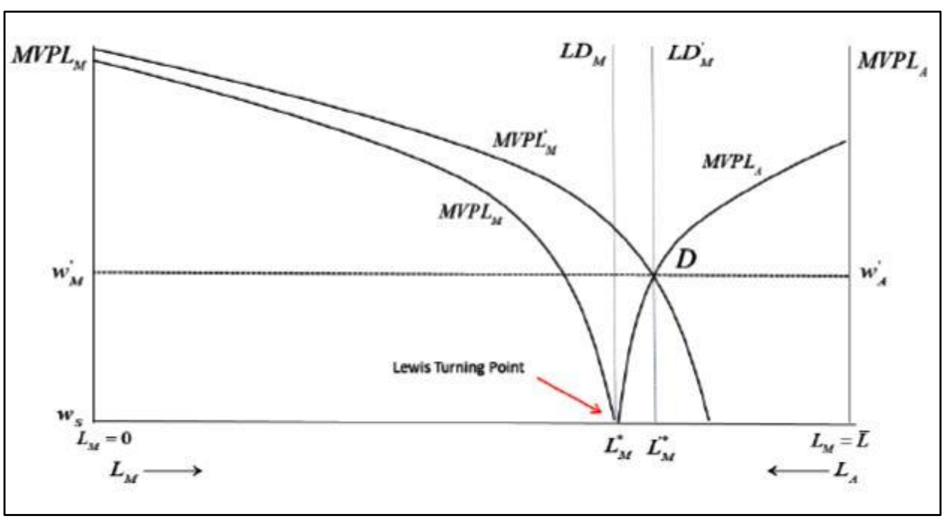
If we have an explicit functional form for the utility function (such as Cobb-Douglas) then equation (3) can be rearranged to obtain an equation that express one of the variables *(X_l or X_0)* in terms of the other. Then, this can be plugged into the budget constraint and rearranged to solve for the optimal amount of X_l and X_0 that the individual will consume. From there, we can back out the total number of hours that she will supply to the labor market H because $H = T - X_l$ where T is the total number of hours available in a day.

The Lewis Model

The Lewis Model is used to model the distribution, accumulation, and growth of capital. We assume that there is a limitless supply of labor in the agricultural (subsistence) sector. Because of this, the marginal product of labor is very small. We assume that $MVPL_A$ (the marginal value product of labor in the agricultural sector) is initially equal to the subsistence wage w_s (which is just enough to keep someone alive). We assume that there are $L_A + L_M = \bar{L}$ people in the economy and that there are two industries (agriculture and manufacturing). Initially everyone works in the agricultural sector such as depicted as point C in the graph below. At some point, capitalists establish a manufacturing sector where they can hire workers at the subsistence wage w_s. They hire workers up to the point where $MVPL_M = w_s$ corresponding to point B (L_M^*) on the graph below. They earn profits which are equal to the area under the $MVPL_M$ curve and above the subsistence wage line.



The capitalists continue to invest their profits in more capital which shifts the $MVPL_M$ curve upward and to the right, and the manufacturing sector employs more and more workers. As the modern sector grows, the demand for food grows, and agricultural producers invest capital into their production as well. This shifts the $MVPL_A$ curve upward and to the left until we get a graph that looks that the one below:



At point L^*_M the wage is still equal to w_s. However, if either the agricultural producers or the urban sector invests more capital, the wage will rise as can be seen by looking at point D on the graph above where the $MVPL'_M$ curve intersects the $MVPL_A$ curve and the wage is w^*_M . This is good for workers as they are able to capture part of the profits of the firms, and their standard of living will rise.

The Harris-Todaro Model

The Harris-Todaro model is used to help us understand migration from rural to urban areas when there continues to be high unemployment rates in urban areas. Let $\overline{w_M}$ be the minimum wage in the manufacturing sector. Let N_M be the number of workers employed in the manufacturing sector, N_U be the number of able-bodied working aged people in the urban sector (some of them may be unemployed), and N_A be the number of workers in the agricultural sector. Let X_A be the amount of the agricultural goods produced and X_M be the amount of the manufactured goods produced.

The minimum manufacturing wage is set by the government, and the wage that an individual can expect to earn by living in the urban area is $w_U^E = \overline{w_M}\,\frac{N_M}{N_U}$ where $\frac{N_M}{N_U}$ is the employment rate in the urban area. That is: the expected wage is the wage in the urban area times the probability of finding a job there. We represent agricultural production by the following function $X_A = q(N_A, \overline{K_A})$ where $\overline{K_A}$ denotes fixed capital. We also assume that $q' > 0$ and $q'' < 0$. By setting the price of the manufactured good equal to 1, we can express the value of the agricultural good as a function of the relative outputs. That is, we can express the normalized price of the agricultural good (p) as follows: $p = \rho\left(\frac{X_M}{X_A}\right)$ where $p' > 0$. That is: if the agricultural good is more scarce than the manufactured good, then the price of the agricultural good will be higher. Recall from the first week of class that the solution to the farmer's problem is characterized by setting MVPL equal to w. Therefore, we can express that solution as follows:

$$W_A = pMP_{N_A} \Rightarrow$$

$$W_A = p\,\frac{\partial q(N_A, \overline{K_A})}{\partial N_A} \Rightarrow$$

$$W_A = \rho\left(\frac{X_M}{X_A}\right)\frac{\partial q(N_A, \overline{K_A})}{\partial N_A}$$

In equilibrium, the wages from the agricultural sector must equal the expected wages in the manufacturing sector (i.e. $W_U^E = W_A$) so the following condition must hold.

$$(4) \quad \overline{w_M}\,\frac{N_M}{N_U} = \rho\left(\frac{X_M}{X_A}\right)\frac{\partial q(N_A, \overline{K_A})}{\partial N_A}$$

To understand the implications of this, let's consider an increase in the minimum wage in the urban sector. This will cause the left side of equation (4) to increase. In order for an equilibrium to be reached after this "shock", several things will occur. First, since the wage in the urban sector is higher, this will lead to fewer workers being hired by firms, so N_M will decrease. Because N_M decreases, there will be fewer manufactured goods produced, so X_M will decrease. Because X_M decreases, the price of the agricultural good $\rho\left(\frac{X_M}{X_A}\right)$ will also decrease. Since the price of the agricultural good will decrease, the number of workers hired in agriculture will decrease which means that they will move into the urban sector which will increase N_U. Therefore, ultimately it is migration from the agricultural sector to the urban sector that help bring the system back into equilibrium (see Chapter 3 appendix for a more rigorous explanation of this).